

2007 FP2 Adapted

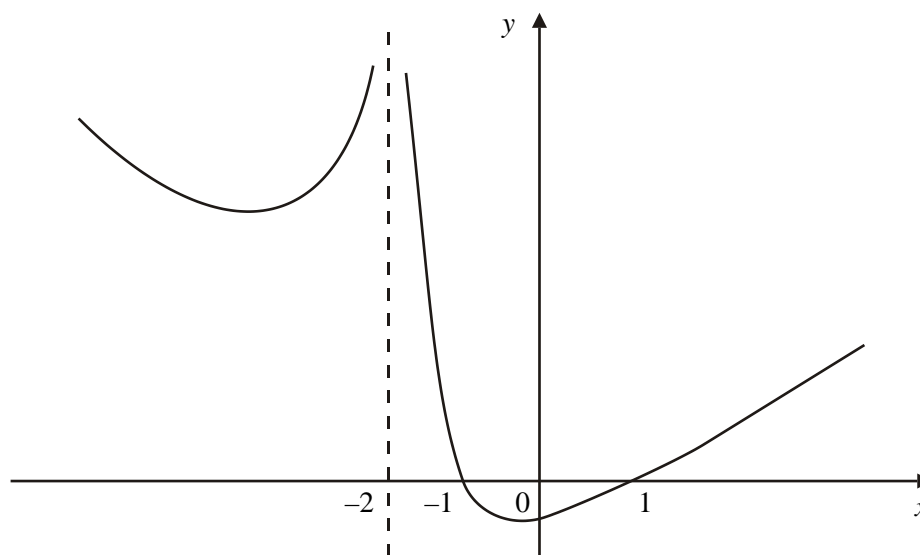
1. Obtain the general solution of the differential equation

$$x \frac{dy}{dx} + 2y = \cos x, \quad x > 0,$$

giving your answer in the form  $y = f(x)$ .

(Total 8 marks)

- 2.



The diagram above shows a sketch of the curve with equation

$$y = \frac{x^2 - 1}{|x + 2|}, \quad x \neq -2.$$

The curve crosses the  $x$ -axis at  $x = 1$  and  $x = -1$  and the line  $x = -2$  is an asymptote of the curve.

- (a) Use algebra to solve the equation  $\frac{x^2 - 1}{|x + 2|} = 3(1 - x)$ .

(6)

(b) Hence, or otherwise, find the set of values of  $x$  for which

$$\frac{x^2 - 1}{|x + 2|} < 3(1 - x).$$

(3)  
(Total 9 marks)

3. A scientist is modelling the amount of a chemical in the human bloodstream. The amount  $x$  of the chemical, measured in  $\text{mg l}^{-1}$ , at time  $t$  hours satisfies the differential equation

$$2x \frac{d^2x}{dt^2} - 6 \left( \frac{dx}{dt} \right)^2 = x^2 - 3x^4, \quad x > 0.$$

(a) Show that the substitution  $y = \frac{1}{x^2}$  transforms this differential equation into

$$\frac{d^2y}{dt^2} + y = 3. \quad \boxed{I}$$

(5)

(b) Find the general solution of differential equation  $\boxed{I}$ .

(4)

Given that at time  $t = 0$ ,  $x = \frac{1}{2}$  and  $\frac{dx}{dt} = 0$ ,

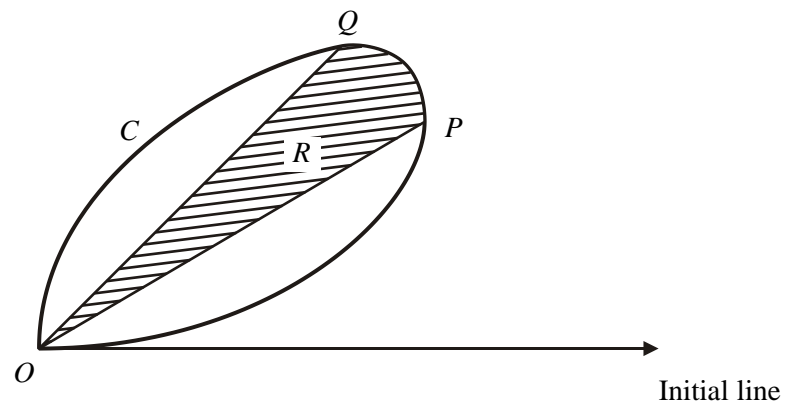
(c) find an expression for  $x$  in terms of  $t$ ,

(4)

(d) write down the maximum value of  $x$  as  $t$  varies.

(1)  
(Total 14 marks)

4.



The diagram above shows a sketch of the curve  $C$  with polar equation

$$r = 4\sin\theta\cos^2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The tangent to  $C$  at the point  $P$  is perpendicular to the initial line.

- (a) Show that  $P$  has polar coordinates  $\left(\frac{3}{2}, \frac{\pi}{6}\right)$ . (6)

The point  $Q$  on  $C$  has polar coordinates  $\left(\sqrt{2}, \frac{\pi}{4}\right)$ .

The shaded region  $R$  is bounded by  $OP$ ,  $OQ$  and  $C$ , as shown in the diagram above.

- (b) Show that the area of  $R$  is given by 
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left( \sin^2 2\theta \cos 2\theta + \frac{1}{2} - \frac{1}{2} \cos 4\theta \right) d\theta$$
 (3)

- (c) Hence, or otherwise, find the area of  $R$ , giving your answer in the form  $a + b\pi$ , where  $a$  and  $b$  are rational numbers. (5)
- (Total 14 marks)**

5. Find the set of values of  $x$  for which

$$\frac{x+1}{2x-3} < \frac{1}{x-3}$$

(Total 7 marks)

6.  $\frac{dy}{dx} - y \tan x = 2 \sec^3 x.$

Given that  $y = 3$  at  $x = 0$ , find  $y$  in terms of  $x$

(Total 7 marks)

7. For the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x(x+3),$$

find the solution for which at  $x = 0$ ,  $\frac{dy}{dx} = 1$  and  $y = 1$ .

(Total 12 marks)

8. (a) Sketch the curve  $C$  with polar equation

$$r = 5 + \sqrt{3} \cos \theta, \quad 0 \leq \theta \leq 2\pi. \quad (2)$$

- (b) Find the polar coordinates of the points where the tangents to  $C$  are parallel to the initial line  $\theta = 0$ . Give your answers to 3 significant figures where appropriate. (6)

- (c) Using integration, find the area enclosed by the curve  $C$ , giving your answer in terms of  $\pi$ . (6)

(Total 14 marks)

9.

$$\frac{dy}{dx} = ye^{x^2}.$$

It is given that  $y = 0.2$  at  $x = 0$ .

- (a) Use the approximation  $\frac{y_1 - y_0}{h} \approx \left(\frac{dy}{dx}\right)_0$ , with  $h = 0.1$ , to obtain an estimate of the value of  $y$  at  $x = 0.1$ .

(2)

- (b) Use your answer to part (a) and the approximation  $\frac{y_2 - y_0}{2h} \approx \left(\frac{dy}{dx}\right)_1$ , with  $h = 0.1$ , to obtain an estimate of the value of  $y$  at  $x = 0.2$ .

Gives your answer to 4 decimal places.

(3)

(Total 5 marks)

10.

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0.$$

At  $x = 0$ ,  $y = 2$  and  $\frac{dy}{dx} = -1$ .

- (a) Find the value of  $\frac{d^3y}{dx^3}$  at  $x = 0$ .

(3)

- (b) Express  $y$  as a series in ascending powers of  $x$ , up to and including the term in  $x^3$ .

(4)

(Total 7 marks)

11. (a) Given that  $z = \cos \theta + i \sin \theta$ , use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2 \cos n\theta. \quad (2)$$

- (b) Express  $32\cos^6\theta$  in the form  $p\cos6\theta + q\cos4\theta + r\cos2\theta + s$ , where  $p, q, r$  and  $s$  are integers. (5)

- (c) Hence find the exact value of

$$\int_0^{\frac{\pi}{3}} \cos^6 \theta d\theta. \quad (4)$$

**(Total 11 marks)**

12. The transformation  $T$  from the  $z$ -plane, where  $z = x + iy$ , to the  $w$ -plane, where  $w = u + iv$ , is given by

$$w = \frac{z+i}{z}, \quad z \neq 0.$$

- (a) The transformation  $T$  maps the points on the line with equation  $y = x$  in the  $z$ -plane, other than  $(0, 0)$ , to points on a line  $l$  in the  $w$ -plane. Find a cartesian equation of  $l$ . (5)

- (b) Show that the image, under  $T$ , of the line with equation  $x + y + 1 = 0$  in the  $z$ -plane is a circle  $C$  in the  $w$ -plane, where  $C$  has cartesian equation

$$u^2 + v^2 - u + v = 0. \quad (7)$$

- (c) On the same Argand diagram, sketch  $l$  and  $C$ . (3)
- (Total 15 marks)**